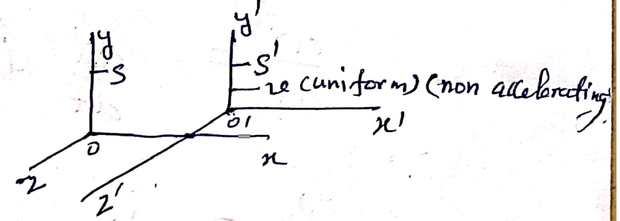
by Dr. Mohammad Aslam

Reference frame :-> A system of co-ordinate which defines the Position of a particle or event in two or three dimensional space is called a frame of reference.

Inertial frame of reference :-> A coordinate system in which Newton's first law of motion holds good is known as inertial frame of reference.

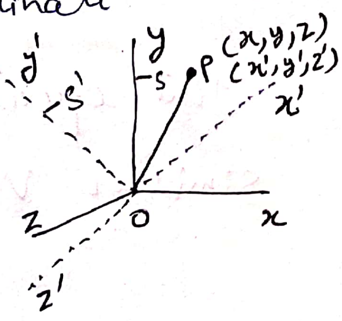
or. A frame of reference moving with uniform velocity w.r.to inertial frame is also known as inertial frame of reference.



Non-Inertial frame of reference :-> An accelorated frame of reference is known as non-inertial frame of reference.

Rotating frame of referencel :->

Let x, y, z represents the cartesian co-ordinate of an inertial frame of refrence (S) and x', y', z' represent the cartesian co-ordinate of a non inertial frame of reference or rotating frame of reference, rotating with angular velocity ω about an axes passing through common origin 'O'.



The position vector of a particle at point 'P' w.r.to S frame is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad \text{--- (1)}$$

The position vector of the particle at point P w.r.to S' frame is

$$\vec{r} = x'\hat{i}' + y'\hat{j}' + z'\hat{k}' \quad \text{--- (2)}$$

differentiating equation (2) w.r.to t

$$\frac{d\vec{r}}{dt} = x'\frac{d\hat{i}'}{dt} + \hat{i}'\frac{dx'}{dt} + y'\frac{d\hat{j}'}{dt} + \hat{j}'\frac{dy'}{dt} + z'\frac{d\hat{k}'}{dt} + \hat{k}'\frac{dz'}{dt}$$

$$\frac{d\vec{r}}{dt} = \frac{dx'}{dt}\hat{i}' + \frac{dy'}{dt}\hat{j}' + \frac{dz'}{dt}\hat{k}' + x'\frac{d\hat{i}'}{dt} + y'\frac{d\hat{j}'}{dt} + z'\frac{d\hat{k}'}{dt}$$

$$\vec{v}_s = \vec{v}_{s'} + x'\frac{d\hat{i}'}{dt} + y'\frac{d\hat{j}'}{dt} + z'\frac{d\hat{k}'}{dt} \quad \text{--- (3)}$$

~~Now~~ we know that

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r} \quad \text{--- (4)}$$

This equation is true for any other value of similar vector so replace $\vec{r}$ to $\hat{i}'$

$$\frac{d\hat{i}'}{dt} = \omega \times \hat{i}', \quad \frac{d\hat{j}'}{dt} = \omega \times \hat{j}', \quad \frac{d\hat{k}}{dt} = \omega \times \hat{k}$$

Now equation (3) becomes,

$$\vec{v}_s = \vec{v}_{s'} + x'(\omega \times \hat{i}') + y(\omega \times \hat{j}') + z'(\omega \times \hat{k}')$$

$$\vec{v}_s = \vec{v}_{s'} + \omega \times (x'\hat{i}' + y'\hat{j}' + z'\hat{k}')$$

$$\vec{v}_s = \vec{v}_{s'} + \omega \times \vec{r} \quad \text{--- (5)}$$

$$\vec{v}_s = \vec{v}_{s'} + \vec{\omega} \times \vec{r} \quad \text{--- (5)}$$

$$\left(\frac{d\vec{r}}{dt}\right)_s = \left(\frac{d\vec{r}}{dt}\right)_{s'} + \vec{\omega} \times \vec{r} \quad \text{--- (6)}$$

Equation (6) is true for any other similar vector quantity so replace $\vec{r}$ to $\vec{v}_s$.

$$\frac{d\vec{v}_s}{dt} = \frac{d\vec{v}_s}{dt} + \vec{\omega} \times \vec{v}_s \quad \text{--- (7)}$$

Put the value of $\vec{v}_s$ in R.H.S of equation (7)

$$\frac{d\vec{v}_s}{dt} = \frac{d}{dt}(\vec{v}_{s'} + \vec{\omega} \times \vec{r}) + \omega \times (\vec{v}_{s'} + \vec{\omega} \times \vec{r})$$

$$\frac{d\vec{v}_s}{dt} = \frac{d\vec{v}_{s'}}{dt} + \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r} + \omega \times \vec{v}_{s'} + \omega \times (\vec{\omega} \times \vec{r})$$

$$\frac{d\vec{v}_s}{dt} = \frac{d\vec{v}_{s'}}{dt} + \vec{\omega} \times \vec{v}_{s'} + \frac{d\vec{\omega}}{dt} \times r + \vec{\omega} \times \vec{v}_{s'} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\frac{d\vec{v}_s}{dt} = \frac{d\vec{v}_{s'}}{dt} + 2\vec{\omega} \times \vec{v}_{s'} + \frac{d\omega}{dt} \times r + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

multiply by m on both side.

$$m\frac{d\vec{v}_s}{dt} = m\frac{d\vec{v}_{s'}}{dt} + 2m(\vec{\omega} \times \vec{v}_{s'}) + m\frac{d\vec{\omega}}{dt} \times \vec{r} + m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{F}_s = \vec{F}_{s'} + 2m(\vec{\omega} \times \vec{v}_{s'}) + m\frac{d\vec{\omega}}{dt} \times \vec{r} + m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{F}_{s'} = \vec{F}_s - 2m(\vec{\omega} \times \vec{v}_{s'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - m\frac{d\vec{\omega}}{dt} \times \vec{r}$$

$$\vec{F}_{s'} = \vec{F}_s + \vec{F}_o$$

where $\vec{F_0} = -2m(\vec{\omega}\times\vec{v_s'}) - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - m\frac{d\vec{\omega}}{dt}\times\vec{r}$

F_0 is called Fictitious force

if ω is constant then $\frac{d\omega}{dt} = 0$

$$\vec{F_0} = -2m(\vec{\omega}\times\vec{v_s'}) - m\vec{\omega}(\omega\times\vec{r})$$

$-2m(\vec{\omega}\times\vec{v_s'}) \longrightarrow$ Coriolis force

$-m\vec{\omega}\times(\vec{\omega}\times\vec{r}) \longrightarrow$ Centrifugal force.

Fictitious force = coriolis force + Centrifugal force,